

## Color test

I just want to be

**sure**

that all my favourite colors

**are**

being displayed correctly on this

**new**

device. If not I'll modify them.

# Inferring cosmic particles

## TEST seminar

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LAL, LRI, University of Paris-Sud XI

February 23rd, 2012

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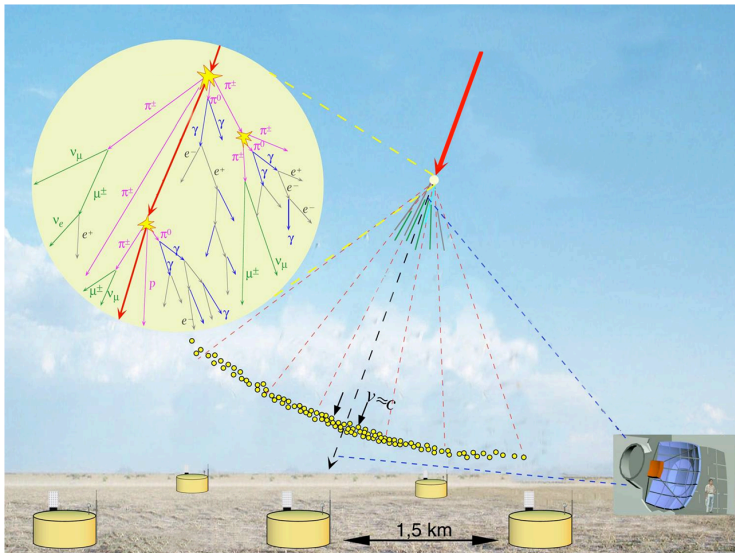
- 1 The Pierre Auger Experiment
- 2 A generative model for the tank signals
- 3 MCMC issues
- 4 The AMOR sampler

## A cosmic shower

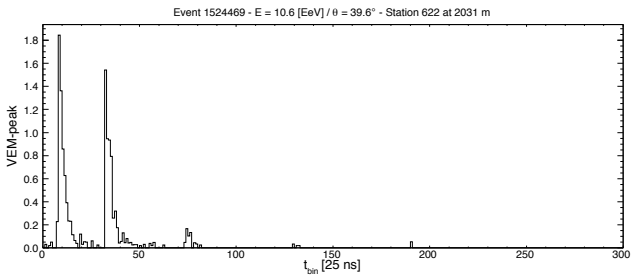
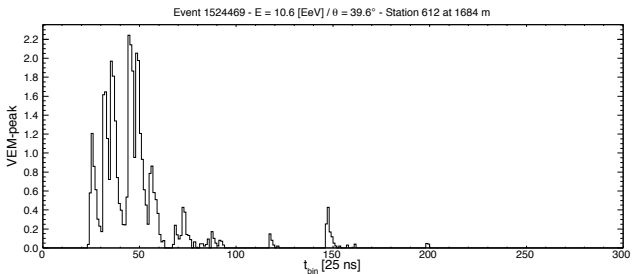


## Google Earth tour

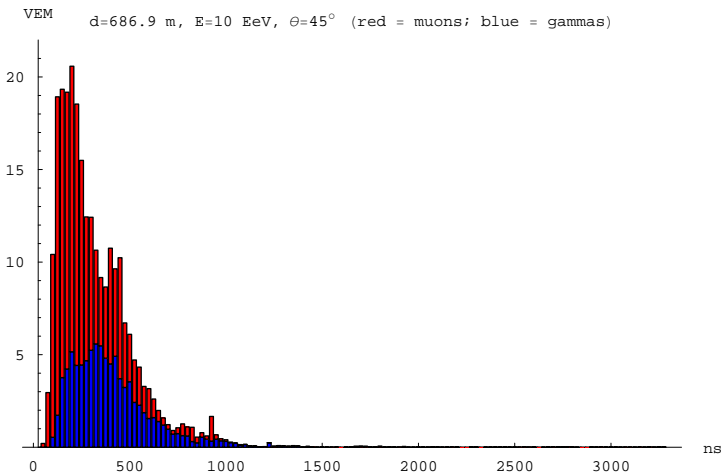
# A cosmic shower (aka flying Greek letters)



# A glance at tank signals

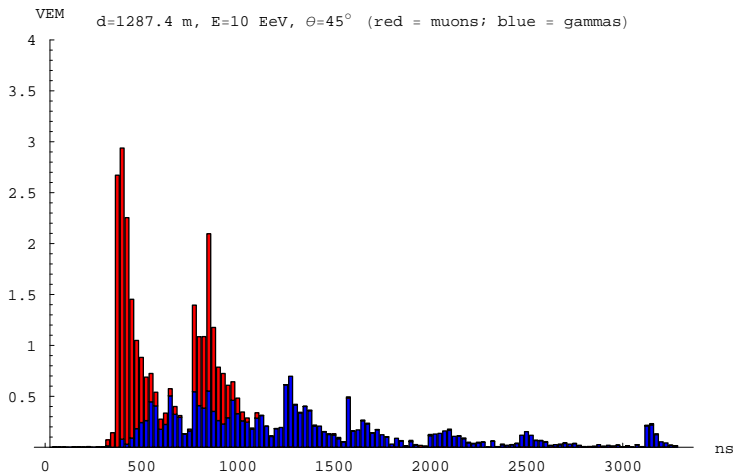


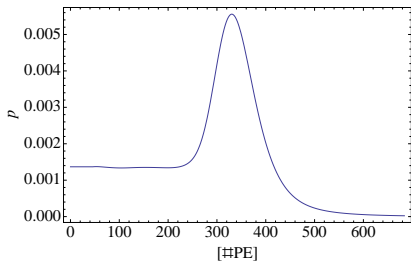
# A glance at tank signals



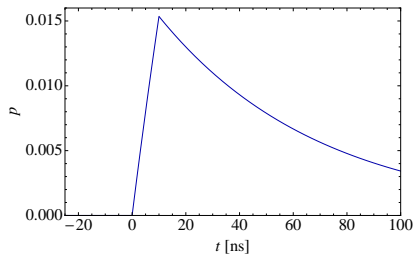


## A glance at tank signals



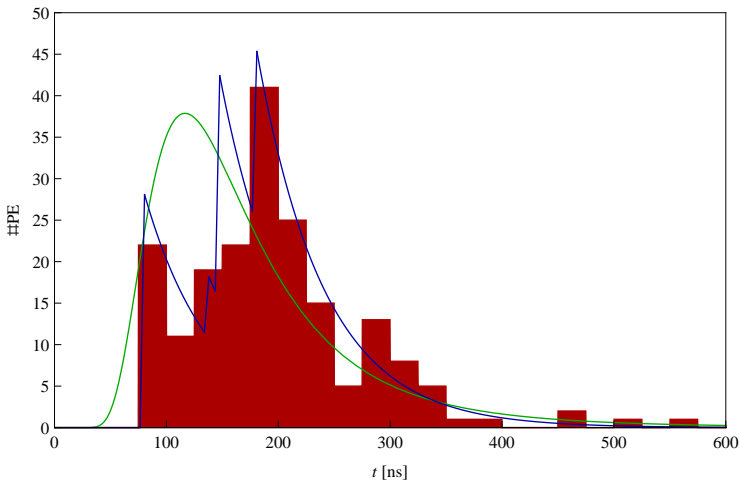


(a) Muonic signal amplitude distribution

(b) Muonic time response model  $p_{\tau, t_d}$ 

### Mean number of Photo-electrons per bin & per muon

$$\bar{n}_i(A_\mu, t_\mu) = A_\mu \int_{t_{i-1}}^{t_i} p_{\tau, t_d}(t - t_\mu) dt.$$



$n_i$  Poisson with mean  $\bar{n}_i(\mathbf{A}_\mu, \mathbf{t}_\mu) = \sum_{j=1}^{N_\mu} \bar{n}_i(A_{\mu j}, t_{\mu j})$ ,

## MCMC issues

- ▶ Possibly high dimensions but also highly correlated model.
  - ▶ Use **adaptive proposals**.
- ▶ The number of muons  $N_\mu$  is unknown.
  - ▶ Use a nonparametric prior or
  - ▶ use a **Reversible Jump** sampler.
- ▶ Likelihood  $\mathcal{P}(\mathbf{n}|\mathbf{A}_\mu, \mathbf{t}_\mu)$  is permutation invariant.
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## Adaptive Metropolis

- ▶ When the target  $\pi(x)$  is multivariate Gaussian with covariance  $\Sigma_\pi$ , the **optimal** choice of  $\Sigma$  is of the order of  $(2.38)^2 \Sigma_\pi / d$   
See Roberts et al. 07
- ▶ You can approximate this with proposals

$$q(\cdot | X_t) = \mathcal{N}(\cdot | X_t, c \Sigma_t)$$

with

$$\Sigma_t = \frac{1}{T} \sum_{i=1}^T (X_i - \bar{X})(X_i - \bar{X})^T.$$

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## Reversible Jump MCMC

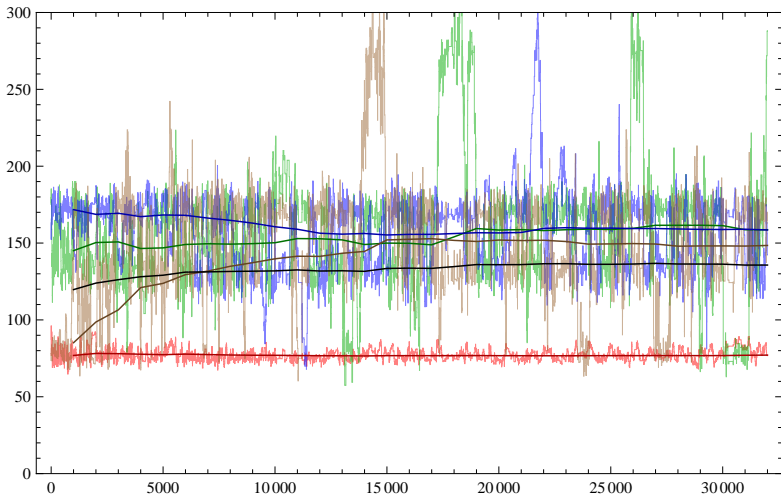
- ▶ Nonparametric prior: needs simple model with **conjugacy** properties for easy sampling, Neal 00.
- ▶ RJMCMC Green 05: MCMC kernel on

$$\bigcup_{N_\mu=1}^{N_\mu^{\max}} \{N_\mu\} \times \{ \text{Parameter space for } N_\mu \text{ muons} \}.$$

- ▶ RJMCMC needs careful **design of transdimensional moves**.

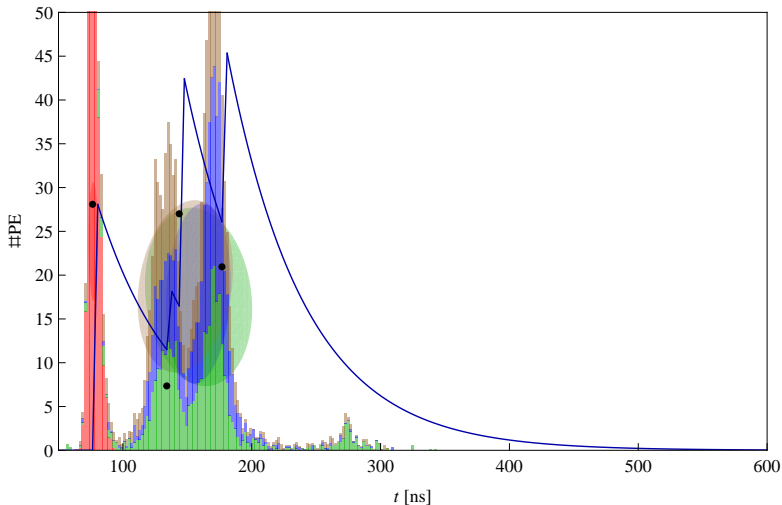
## Label-Switching

- ▶ If the prior is **exchangeable**, then  $\pi(\mathbf{A}_\mu, \mathbf{t}_\mu)$  is permutation-invariant.



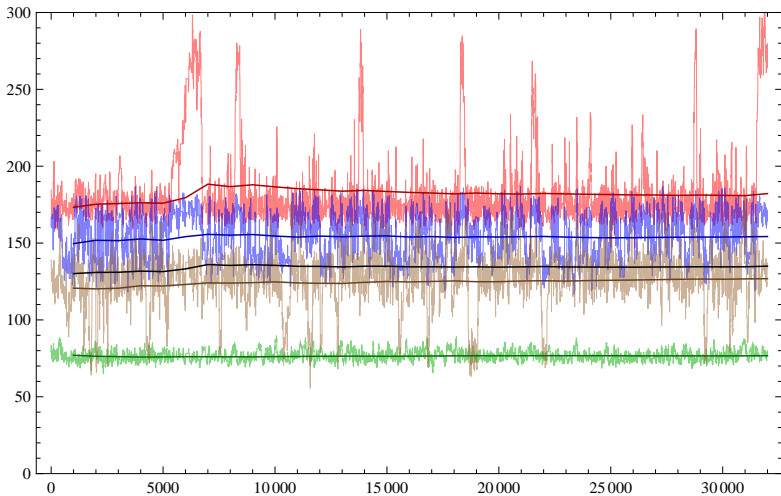
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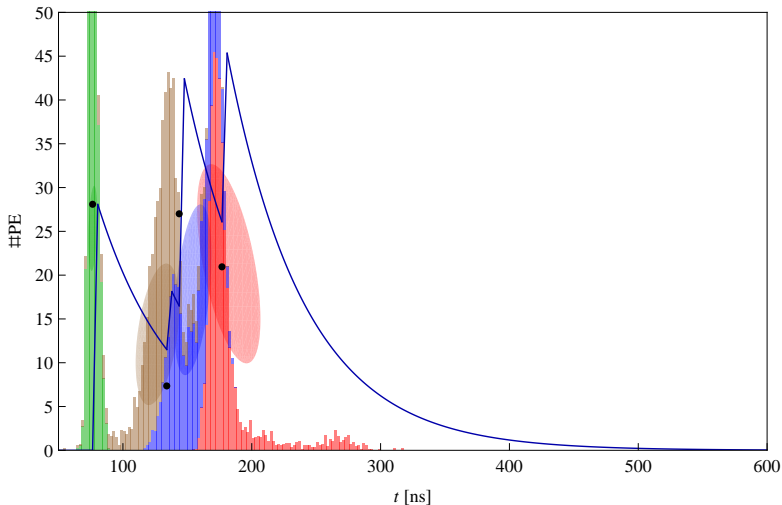
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## The AMOR sampler

```

AMOR( $\pi(x), X_0, T, \mu_0, \Sigma_0, c$ )
1    $S \leftarrow \emptyset$ 
2   for  $t \leftarrow 1$  to  $T$ 
3        $\Sigma \leftarrow c\bar{\Sigma}_{t-1}$   $\triangleright$  scaled adaptive covariance
4        $\tilde{X} \sim \mathcal{N}(\cdot | X_{t-1}, \Sigma)$   $\triangleright$  proposal
5        $\tilde{P} \sim \arg \min_{P \in \mathfrak{P}} L_{(\mu_{t-1}, \Sigma_{t-1})}(P\tilde{X})$   $\triangleright$  pick an optimal permutation
6        $\tilde{X} \leftarrow \tilde{P}\tilde{X}$   $\triangleright$  permute
7       if  $\frac{\pi(\tilde{X}) \sum_{P \in \mathfrak{P}} \mathcal{N}(PX_{t-1} | X, \Sigma)}{\pi(X_{t-1}) \sum_{P \in \mathfrak{P}} \mathcal{N}(PX | X_{t-1}, \Sigma)} > \mathcal{U}[0, 1]$  then
8            $X_t \leftarrow X$   $\triangleright$  accept
9       else
10           $X_t \leftarrow X_{t-1}$   $\triangleright$  reject
11           $S \leftarrow S \cup \{X_t\}$   $\triangleright$  update posterior sample
12           $\mu_t \leftarrow \mu_{t-1} + \frac{1}{t}(X_t - \mu_{t-1})$   $\triangleright$  update running mean and covariance
13           $\Sigma_t \leftarrow \Sigma_{t-1} + \frac{1}{t}((X_t - \mu_{t-1})(X_t - \mu_{t-1})^\top - \Sigma_{t-1})$ 
14  return  $S$ 

```

## (Scientific) conclusion (and ads)

- ▶ Particle Physics is cool,
- ▶ MCMC is neat,
- ▶ Mixing the two is great.
- ▶ To learn more on cosmic rays and Auger: [Karim Louedec's thesis \(link\)](#).
- ▶ On the model & AMOR: our last [AISTATS paper \(link\)](#) and a submitted Physics paper I can probably send you.
- ▶ On adaptive MCMC: see [Fort et al 2011 \(link\)](#)
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