Multiresolution Analysis of Incomplete Rankings

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Abstract

Incomplete rankings on a set of items $\{1, \dots, n\}$ are orderings of the form $a_1 < \cdots < a_k$, with $\{a_1, \dots, a_k\} \subset \{1, \dots, n\}$ and $k < n$. Though they arise in many modern applications, only a few methods have been introduced to manipulate them, most of them consisting in representing any incomplete ranking by the set of all its possible linear extensions on $\{1, \dots, n\}$. In this talk, I will introduce a completely novel approach, which allows to treat incomplete rankings directly, representing them as injective words over $\{1, \dots, n\}$. Unexpectedly, operations on incomplete rankings have very simple equivalents in this setting and the topological structure of the complex of injective words can be interpretated in a simple fashion from the perspective of ranking. We exploit this connection here and use recent results from algebraic topology to construct a multiresolution analysis and develop a wavelet framework for incomplete rankings. Though purely combinatorial, this construction relies on the same ideas underlying multiresolution analysis on a Euclidean space, and permits to localize the information related to rankings on each subset of items. It paves the way for many statistical applications, including preference data analysis and the design of recommender systems